Orange Board of Education
Pre-Calculus

Cycle1: Functions, Polynomial, & Rational

ORANGE PUBLIC SCHOOLS 2015 - 2016

OFFICE OF CURRICULUM AND INSTRUCTION

OFFICE OF MATHEMATICS

Cycle I:

Topic: Units 1-2. Functions, Polynomial and Rational	**Skills:** • Determine if a relationship represents a function • Evaluate the value of a function • Determine the domain and range of a function • Evaluate, oven a given interval, the average change of a function • Determine intervals where a function is increasing, decreasing, and constant • Determine the inverse of a function, and whether a function is 1 to 1 • Evaluate combinations of and composite functions • Determine if a function is odd, even, or neither, both graphically and algebraically • Graph the 8 basic parent functions • Graph, read, and evaluate piecewise-defined functions • Understand how transformations are represented, both in equations and in graphs • Sketch functions based off their parent functions and corresponding transformations • Identify key characteristics of parent functions, using domain, range, maxima and minima, and intervals of increasing and decreasing • Solve real-world problems using a variety of functions • Examine equations of polynomial functions to determine left and right end behavior • Identify the zeros of a polynomial function and its multiplicity • Identify the domain, range, and degree of polynomial functions • Analyze the graphs of polynomial functions with respect to turning points, zeros, and end behavior • Form polynomials from zeros and graphs • Divide polynomials with long and synthetic division • Compare and contrast the properties of real and imaginary numbers • Perform arithmetic operations on complex numbers. • Find the real and complex zeros of a polynomial • Find the domain, and the vertical and horizontal asymptotes of a rational function	**Projected # of days:** 45 (23 block)

Unit 1—Functions and Their Graphs

Goal(s)(NJCCCS and CCSS):

F.BF.1. Write a function that describes a relationship between two quantities.
F.BF.4. Find inverse functions.
F.BF.4b. Verify by composition that one function is an inverse of another.
F.BF.4c. Read values of an inverse function from a graph or table, given that the function has an inverse.
F.BF.4d. Produce an invertible function from a non-invertible function by restricting a domain.
F.IF.7. Graph functions expressed symbolically and show key features of the graph, by hand in the simple cases and using technology for more complicated cases.

Essential Questions:

How are functions and their graphs related?
What are some properties and patterns of functions and their related parent functions?
How do patterns and functions help us describe data and real-world physical phenomena?

Skills/Knowledge/Understandings:

1. Determine if a relationship represents a function
2. Evaluate the value of a function
3. Determine the domain and range of a function
4. Evaluate, oven a given interval, the average change of a function
5. Determine intervals where a function is increasing, decreasing, and constant
6. Determine the inverse of a function, and whether a function is 1 to 1
7. Evaluate combinations of and composite functions
8. Determine if a function is odd, even, or neither, both graphically and algebraically
9. Graph the 8 basic parent functions
10. Graph, read, and evaluate piecewise-defined functions
11. Understand how transformations are represented, both in equations and in graphs
12. Sketch functions based off their parent functions and corresponding transformations
13. Identify key characteristics of parent functions, using domain, range, maxima and minima, and intervals of increasing and decreasing
14. Solve real-world problems using a variety of functions

Objectives:

1) Using their equations, SWBAT recognize functions and determine its domain, range, and average change over an interval
2) Using their graphs, SWBAT determine parent functions and transform graphs of parents functions
3) Using equations of multiple functions, SWBAT form combinations of functions and composite functions
4) Using its definition, SWBAT define, find, and verify inverse functions

5) Using their equations, SWBAT solve everyday problems that can be modeled using functions

Assessments:

Formative:	Summative: Unit 1 exam, involving functions and graphs, operations on functions, inverse functions.	Authentic:
Daily exit slips, always including at least one question that requires students to summarize and write, in their own words, what they learned that day.		Choose a physical phenomenon that is represented by one of the parent functions. Graph it, denote the domain and range, and find the transformations needed to go from the parent function to this function. Additionally, students need to clearly define what each variable represents in the physical world.

Literacy Connections:

Every exit slip will require students to synthesize their daily learning and write, in their own words, what they learned.

Interdisciplinary Connections:

Physics—Modeling projectile motion
Physics—Using Hooke's Law of springs
History—Using regression models for prediction and analysis
Social Science—Predicting future US census data

Technology Integration:

TI-84 for plotting the general form of the parent functions
TI-84 for graphing functions and finding the zeros
Smart Board for showing visually how function transformations affect graphs.

Key Vocabulary:

Function
Domain
Range
Implicit Domain
Inverse Function
1-to-1
Parent Function
Composite Function
Piecewise Function
Transformations
Symmetry

Odd/even functions
Increasing/decreasing
Relative extrema
Rate of change

Useful Sites:
Functions and Domain/Range--http://www.purplemath.com/modules/fcns2.htm
Piecewise Functions-- http://www.mathsisfun.com/sets/functions-piecewise.html
Composition of Functions--http://www.purplemath.com/modules/fcncomp.htm
Function Transformations--http://math.kennesaw.edu/~sellerme/sfehtml/classes/math1113/transformation.pdf
Inverse Functions--http://www.purplemath.com/modules/invrsfcn.htm
Lesson Reviews--http://www.khanacademy.org

Primary Documents:

Text Crosswalk:
Larson and Hostetler, Brooks/Cole, 7th edition. 2007.
Unit 1 covers pages 1-126.
Page 70 shows graphs of all 8 of the most common parent functions.
The front inside cover of the book shows graphs and properties of the 8 parent functions, as well as of transcendental functions.
A checklist summary of Unit 1 objectives is found on page 116.
An extensive set of review problems is found on pages 117-122.
Page 123 offers a sample Unit Assessment.
Page 124 offers a proof of the midpoint formula.
Pages 125-126 offer challenging problems that require extra critical thinking from students dealing with Unit 1.

*Differentiation: www.marzanoresearch.com/free_resources/itembank.aspx

Unit 2—Polynomial and Rational Functions

Goal(s)(NJCCCS and CCSS):

A.APR.6. Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x)+r(x)/b(x), where a(x), b(x), q(x) and r(x) are polynomials with the degree of r(x) less than the degree of b(x) using inspection, long division, or, for the more complicated examples, a computer algebra system.

A.APR.7. Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication and division by a nonzero rational expression; add, subtract, multiply and divide rational expressions.

F.IF.7. Graph functions expressed symbolically and show key features of the graph, by hand in the simple cases and using technology for more complicated cases.

F.IF.7d. Graph rational functions, identifying zeros when suitable factorizations are available, and showing end behavior.

N.CN.3. Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

N.CN.8. Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$.

N.CN.9. Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Essential Questions:

What are common characteristics and properties of polynomials?

How do we go about efficiently graphing polynomials?

How do patterns and polynomials help us describe data and physical phenomena and solve a variety of problems?

What can asymptotes tell us about functions and their behavior?

Skills/Knowledge/Understandings:

1) Examine equations of polynomial functions to determine left and right end behavior
2) Identify the zeros of a polynomial function and its multiplicity
3) Identify the domain, range, and degree of polynomial functions
4) Analyze the graphs of polynomial functions with respect to turning points, zeros, and end behavior
5) Form polynomials from zeros and graphs
6) Divide polynomials with long and synthetic division
7) Compare and contrast the properties of real and imaginary numbers
8) Perform arithmetic operations on complex numbers
9) Find the real and complex zeros of a polynomial
10) Find the domain, and the vertical and horizontal asymptotes of a rational function

Objectives:

1) Using polynomials, SWBAT define rational expressions and divide polynomials to create rational expressions
2) Using its equation, SWBAT determine the zeros (real and complex) of a polynomial
3) Using the equations of rational functions, SWBAT find their intercepts, asymptotes, holes, domain, and range

4) Using their two components, SWBAT perform arithmetic operations on complex numbers

Assessments:

Formative:	Summative: Chapter 2 assessment, including end behavior, finding zeros (real and complex), complex operations, the Fundamental Theorem of Algebra, and graphing polynomials.	Authentic:
Daily exit slips, always including at least one question that requires students to summarize and write, in their own words, what they learned that day.		Create-a-graph. Pick a real-world situation that is modeled by a polynomial or rational function. Then graph it by hand, taking into account the end behavior, zeros, multiplicity, and number of turning points. Label the axis and clearly define what they mean in this specific context.

Literacy Connections:

Students will read an article on population modeling and how their new knowledge can help model and predict future population sizes.
Students will be able to compare and contrast properties of real and imaginary numbers.
Every exit slip will require students to synthesize their daily learning and write, in their own words, what they learned.

Interdisciplinary Connections:

Business-Manufacturing Predictions
History—Population Modeling
Science—Boyle's Law, photosynthesis

Technology Integration:

TI-84 to graph polynomials and verify solutions
TI-84 to approximate the real zeros of a polynomial
Smart Board for visually showing horizontal and vertical asymptotes.

Key Vocabulary:

Polynomial
Degree
Zeros
Multiplicity
Higher-order polynomial
Leading coefficient
Complex numbers
Complex conjugate
Horizontal asymptote
Vertical asymptote
Real and imaginary components

Fundamental Theorem of Algebra Rational Function
Useful Sites: Complex numbers--http://www.purplemath.com/modules/complex.htm Complex numbers video-- https://www.khanacademy.org/math/algebra/complex-numbers/complex_numbers/v/complex-numbers--part-1 End behavior-- http://www.purplemath.com/modules/polyends.htm Fundamental Theorem of Algebra-- http://www.mathsisfun.com/algebra/fundamental-theorem-algebra.html How to Find Asymptotes-- http://www.coolmath.com/precalculus-review-calculus-intro/precalculus-algebra/18-rational-functions-finding-horizontal-slant-asymptotes-01.htm Lesson Reviews--http://www.khanacademy.org
Primary Documents:
Text Crosswalk: *Larson and Hostetler, Brooks/Cole, 7th edition. 2007.* Unit 2 covers pages 127-216. A checklist summary of Unit 2 objectives is found on page 207. A comprehensive set of review problems can be found on pages 208-211. A sample Unit Assessment can be found on page 212. Page 213 offers proofs of the Remainder and Factor Theorems. Page 214 offers proofs of the Linear Factorization Theorem and the Factors of a Polynomial. Pages 215-216 offer challenging problems that require extra critical thinking from students dealing with Unit 1.

*Differentiation: www.marzanoresearch.com/free_resources/itembank.aspx